Numerical modal analysis in dispersive and dissipative plasmonic structures.

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Abstract—Modal analysis is an essential tool since it straightly provides the lighting conditions under which a plasmonic structure can “sing”. Modes appear as solutions of source free Maxwell’s equations. For dispersive and dissipative structures, the associated spectral problem is not standard, being generally non-linear in frequency and not self-adjoint. We developed and implemented two finite element formulations to tackle this non-standard eigenvalue problem in a 2D multi-domain closed cavity. Their numerical validity are checked against the analytical solution.

1. INTRODUCTION TO SPECTRAL ANALYSIS OF MAXWELL’S OPERATOR

We start with Maxwell’s equations free of electromagnetic sources. The problem is considered in the time-harmonic regime $\exp(-i\omega t)$ where $\omega$ is the frequency of the electromagnetic field. We will restrict our studies to linear, homogeneous and isotropic materials. In this frame, the electric field $E(\omega, x)$ is the solution of

$$\nabla \times \mu(\omega, x)^{-1} \nabla \times E(\omega, x) = \omega^2 \varepsilon_0 \mu_0 \varepsilon(\omega, x) E(\omega, x),$$

where $\varepsilon$ is the relative permittivity of the different considered media, and $\mu$ the relative permeability ($\varepsilon_0$ and $\mu_0$ being respectively the vacuum permittivity and permeability). Without presence of dispersion, the spectral analysis of the operators involve in Eq. 1 leads to a classical linear eigenvalue problem (EVP) since the relative permeability and permittivity tensors do not depend on $\omega$. However, when dispersion occurs, this spectral analysis is much more difficult to perform.

2. RELATIVE PERMITTIVITY MODEL

Photonic structures based on plasmonic resonances classically involves metallic materials whose dielectric permittivity $\varepsilon$ dependency on the electromagnetic wave frequency $\omega$ can be approached by a sum of Drude-Lorentz resonances. An accurate model for the optical range can be found in [1] and consists in the combination of two contributions: The Drude resonance model and the Drude-Lorentz resonance model:

$$\varepsilon_{DL}(\omega) = \varepsilon_{\infty} - \frac{\omega_D^2}{\omega [\omega + i\gamma_D]} - \frac{\Delta \varepsilon \omega_L^2}{\omega^2 + i\gamma_L \omega - \omega_L^2},$$

where the physical meaning of the different parameters of Eq. 2 can be found in [1].

The dielectric permittivity model described in Eq. 2 is the starting point of our two different modal analysis formulations. Note that the considered media in the present case are non-magnetic, hence $\mu(\omega, x) = I_d$, where $I_d$ is the identity tensor.

3. SPECTRAL ANALYSIS BASED ON POLYNOMIAL EVP

By injecting the relative permittivity model of Eq. 2 in Eq. 1, one obtains the following equation:

$$\nabla \times \nabla \times E = \omega^2 \varepsilon_0 \mu_0 \left[ \varepsilon_{\infty} - \frac{\omega_D^2}{\omega [\omega + i\gamma_D]} - \frac{\Delta \varepsilon \omega_L^2}{\omega^2 + i\gamma_L \omega - \omega_L^2} \right] I_d E.$$

This formulation leads to a polynomial EVP which is tractable, thanks to recent advances in generalized polynomial eigenvalue solver [2], through a finite element formulation [3].
4. SPECTRAL ANALYSIS BASED ON THE RESONANCE FIELD FORMALISM

This spectral analysis is based on the auxiliary field formalism introduced by A. Tip in 1997 [4, 5]. By the introduction of additional fields, so called “auxiliary fields”, to the classical electromagnetic fields, the classical Maxwell operator can be extended to a linear generalized one. However, we use a slightly different formalism, denominated as “resonance formalism”, especially adapted to the dispersive model described by Eq. 2. With these auxiliary fields and resonance formalism, we can construct an extended standard (linear) EVP to consider dispersive and dissipative systems.

5. NUMERICAL VALIDATION ON A CLOSED DISPERSIVE CAVITY

The two previously described formulations were implemented [6] for 2D perfectly conducting multi-domain cavities including Gold. Excellent agreement is shown between the complex eigenfrequencies obtained with the different methods (see Fig 1. for an example in a particular case).

![Figure 1: Real part and imaginary part of the eigenfrequencies obtained analytically, with the polynomial formulation and the auxiliary fields formulation for a 2D perfectly conducting closed cavity filled with Gold.](image)

The proposed methods can be applied to more realistic configurations like open electromagnetic systems thanks to the introduction of perfectly matched layers [7].

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