

## Analysis of Planar Structure with Patch Superconductor Material and PBG Substrate

Humberto Fernandes\*, Hugo Maia and Leonardo Caetano

Department of Electrical Engineering , Federal University Rio Grande do Norte, 59078-970 - Natal-RN, Brazil

\*corresponding author [humbeccf@ct.ufrn.br](mailto:humbeccf@ct.ufrn.br), [hummychel@yahoo.com.br](mailto:hummychel@yahoo.com.br), [leoteleco@yahoo.com.br](mailto:leoteleco@yahoo.com.br)

**Abstract** — The analysis of the resonance frequency, efficiency, quality factor and pattern fields of microstrip antennas array, with superconductor patch for high critical temperatures, and PBG (Photonic Band Gap) substrate, are presented. The concise full wave Transverse Transmission Line (TTL) method is used in the analysis. New results of the resonance frequency, efficiency, quality factor and pattern fields of microstrip antennas array are presented.

**Keywords** - temperature; antenna; microstrip; photonic; substrate; efficiency.

### I. INTRODUCTION

H. R. Onnes, in 1908, discovered that basic metals had null resistance, when the temperature was below of the critical temperature,  $T_c$ . In 1933, W. H. Meissner and R. Ochsenfeld reported that in type I superconductors, when they were caught a cold below the critical temperature, the magnetic flow was expelled of the interior of the superconductor [1] and [2]. This phenomenon is known as the Meissner effect, and it is illustrated in Figure 1

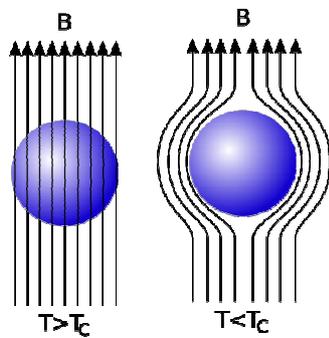


Figure 1. Behavior of superconductor in the presence of a magnetic field  $B$ , the Meissner effect.

In this work microstrip antennas array using superconductors patch and photonic band gap substrate are used. At superconductors, below of the critical temperature, the magnetic flow is expelled of the interior of the superconductor, phenomenon known as Meissner effect [1]-[3].

The substrate has an important role in the performance of the structure. One of the main effects of the substrate is to increase the bandwidth and efficiency. Dielectric materials can

be used with and without losses, semiconductors, ferrites, PBG, among others.

The type of substrate changes the parameters of the antenna. PBG (Photonic Band Gap) materials can be used as substrates, to improve air irradiation, thus reducing the occurrence of surface waves and the resulting diffraction edge , responsible for radiation pattern deterioration [4] - [5].

In microwave applications, there is a predominance of two-dimensional structures (crystals), as illustrated in Figure 2.

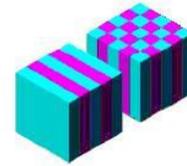


Figure 2. One and Two-dimensional Periodic Structures.

The resonant frequency and pattern fields were presented recently by the authors in [6], for microstrip antennas array with very high superconductor temperature [3],[7]-[8]. In this work the analysis and new results of the efficiency and quality factor are presented. The structure of one element antenna is shown in Fig. 3.

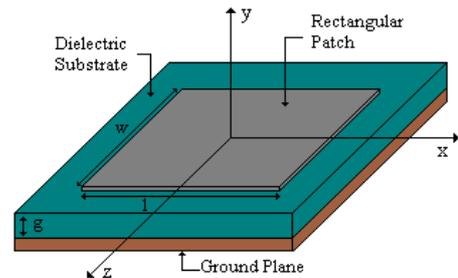


Figure 3. Photonic-superconducting microstrip antenna with patch of width,  $w$ , thickness,  $g$ , and length,  $l$ .

Considering the microstrip antenna resonator of Fig. 2, the equations that represent the electromagnetic fields in the  $x$  and  $z$  directions as function of the electric and magnetic fields in the  $y$  direction are obtained, applying the TTL method [6],[8].

## II. THEORY

### A. TTL Analysis

In the TTL analysis, using the Maxwell's equations, after various algebraic manipulations, the general equations for the antenna are obtained, in the FTD - Fourier Transformed Domain, for the x and z directions as functions of the y variable [6],[8]:

The solutions of the electromagnetic field in y,  $\tilde{E}_y$  and  $\tilde{H}_y$  are obtained through the solutions of the Helmholtz wave equations [15], in the spectral domain, where:

For Region 1:

$$\tilde{E}_{x1} = \frac{1}{\gamma_1^2 + k_1^2} [-j\alpha_n \gamma_1 A_{1e} \sinh(\gamma_1 y) + \omega \mu \beta_k A_{1h} \sinh(\gamma_1 y)] \quad (1)$$

$$\tilde{E}_{z1} = \frac{1}{\gamma_1^2 + k_1^2} [-j\beta_k \gamma_1 A_{1e} \sinh(\gamma_1 y) - \omega \mu \alpha_n A_{1h} \sinh(\gamma_1 y)] \quad (2)$$

$$\tilde{H}_{x1} = \frac{1}{\gamma_1^2 + k_1^2} [-j\alpha_n \gamma_1 A_{1h} \cosh(\gamma_1 y) - \omega \epsilon_1 \beta_k A_{1e} \cosh(\gamma_1 y)] \quad (3)$$

$$\tilde{H}_{z1} = \frac{1}{\gamma_1^2 + k_1^2} [-j\beta_k \gamma_1 A_{1h} \cosh(\gamma_1 y) + \omega \epsilon_1 \alpha_n A_{1e} \cosh(\gamma_1 y)] \quad (4)$$

For Region 2:

$$\tilde{E}_{x2} = \frac{1}{\gamma_2^2 + k_2^2} [j\alpha_n \gamma_2 A_{2e} e^{-\gamma_2(y-h)} + \omega \mu \beta_k A_{2h} e^{-\gamma_2(y-h)}] \quad (5)$$

$$\tilde{E}_{z2} = \frac{1}{\gamma_2^2 + k_2^2} [j\beta_k \gamma_2 A_{2e} e^{-\gamma_2(y-h)} - \omega \mu \alpha_n A_{2h} e^{-\gamma_2(y-h)}] \quad (6)$$

$$\tilde{H}_{x2} = \frac{\gamma_2 + k_2}{\gamma_2^2 + k_2^2} [j\alpha_n \gamma_2 A_{2h} e^{-\gamma_2(y-h)} - \omega \epsilon_2 \beta_k A_{2e} e^{-\gamma_2(y-h)}] \quad (7)$$

$$\tilde{H}_{z2} = \frac{1}{\gamma_2^2 + k_2^2} [j\beta_k \gamma_2 A_{2h} e^{-\gamma_2(y-h)} + \omega \epsilon_2 \alpha_n A_{2e} e^{-\gamma_2(y-h)}] \quad (8)$$

The constants are found and the results are:

$$A_{1e} = \frac{j(\alpha_n \tilde{E}_{xh} + \beta_k \tilde{E}_{zh})}{\gamma_1 \sinh(\gamma_1 y)} \quad (9)$$

$$A_{1h} = \frac{\beta_k \tilde{E}_{xh} - \alpha_n \tilde{E}_{zh}}{\omega \mu \sinh(\gamma_1 y)} \quad (10)$$

$$A_{2e} = -\frac{j(\alpha_n \tilde{E}_{xh} + \beta_k \tilde{E}_{zh})}{\gamma_2} \quad (11)$$

$$A_{2h} = \frac{\beta_k \tilde{E}_{xh} - \alpha_n \tilde{E}_{zh}}{\gamma_2} \quad (12)$$

where i = 1, 2 are the dielectric regions of structure,  $\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2$  is the propagation constant in y direction,  $\alpha_n$  is spectral variable in x direction,  $\beta_k$  is spectral variable in z

direction,  $k_i^2 = \omega^2 \mu \epsilon = k_0^2 \epsilon_{ri}^*$  is the wave number of the dielectric region, and  $\epsilon_{ri}^* = \epsilon_{ri} - j \frac{\sigma_i}{\omega \epsilon_0}$  is the relative dielectric permittivity of the complex material.

After the application of the boundary conditions [9]-[10], the dyadic Z matrix is obtained, and adequate basis functions are used to substitute the current densities. The electric fields are zero in the patch.

$$\begin{bmatrix} Z_{xx} - Z_S & | & Z_{xz} \\ \hline Z_{zx} & | & Z_{zz} - Z_S \end{bmatrix} \cdot \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix} = \begin{bmatrix} \tilde{E}_x^{out} \\ \tilde{E}_z^{out} \end{bmatrix} \quad (13)$$

$$\text{Where,} \quad Z_S = \frac{1}{\sigma_s t} \quad (14)$$

$Z_S$  is the superconductor impedance,  $\sigma_s$  is the conductivity and "t" is the thickness of the superconductor patch.

The Moment method is used to eliminate the electric fields in (13), and to obtain the homogeneous matrix equation (15) for the calculation of the complex resonant frequency.

$$\begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (15)$$

The determinant of this K matrix, which is the inner product of the Z matrix and the basis functions, provides the real and imaginary resonant frequencies, which are calculated using computational programs developed by the authors, in Fortran Power Station Language.

### B. BCS Theory

The quantum theory of the superconductivity was launched in 1957 by the work of Bardeen, Cooper and Schrieffer [11]. This theory include:

a) An attractive interaction between electrons can be conducted to a ground state, separated from an excited states by an energy gap, that separates the superconducting electrons below of the gap of the normal electrons. The critical field, the thermal and electromagnetic properties are many other consequences of this energy gap.

b) The penetration depth  $\lambda_L$  appear as natural consequences of the BCS theory [3]. The London equation is obtained at the magnetic fields, that vary slowly in space. Thus the Meissner effect is obtained naturally.

The London equations are used [11],

$$e \vec{E} = m \frac{d \vec{v}}{dt} \quad (16)$$

$$\Lambda \frac{\partial \vec{j}}{\partial t} = \vec{E} \quad (17)$$

Where,  $\Lambda$  is London constant,  $m$  is the electrons number,  $e$  is the electron charge and  $v$  is the Fermi velocity

### III. ANTENNA ARRAYS, QUALITY FACTOR AND EFFICIENCY

The antenna array [12] consists of a finite number of identical irradiant elements, which combines the induced signals in these antennas, to form the array. The maximum beam direction is controlled, adjusting the phase of the sign in elements of different spaces. The phase induced in the several adjustments in the elements, so that the sign obtain maximum directivity and gain.

The antenna array can be classified as linear and planar. The planar array of microstrip antenna has the array factor,

$$FA_n = \left[ \frac{\text{sen}\left(\frac{N}{2}\psi\right)}{\left(\frac{N}{2}\right)} \right] \quad (18)$$

$$\text{Where } \psi = kd \cos \theta + \beta \quad (19)$$

The quality factor [13] is a merit figure that represents the antenna losses. Typically there are radiations, conductor, dielectric and surface wave losses. Therefore the total quality factor  $Q_t$  is influenced by all of these losses and is, in general, written as:

$$\frac{1}{Q_t} = \frac{1}{Q_{rad}} + \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_{sw}} \quad (20)$$

Where  $Q_t$  = total quality factor;  $Q_{rad}$  = quality factor due to radiation (space wave) losses;  $Q_c$  = quality factor due to conduction losses (ohmic);  $Q_d$  = quality factor due to dielectric losses;  $Q_{sw}$  = quality factor due to surface waves, and in this work no losses was considered, for superficial waves,  $Q_{sw}=1$ , expressed as [11],

$$Q_c = g\sqrt{\pi f \mu \sigma} \quad (21)$$

$$Q_d = \frac{1}{\tan \delta} \quad (22)$$

$$Q_{rad} = \frac{2\omega\epsilon_r}{g \cdot \frac{G_t}{l}} \cdot \frac{\iint |E|^2 dA}{\iint |E|^2 dl} \quad (23)$$

Where  $G_t$  is the conductance,  $\sigma$  is the conductivity,  $E$  is the electric field, and  $g$  is the substrate thickness.

The efficiency is the ratio of the quality factor total [13] by the quality factor of radiation as [14]-15]:

$$\eta = \frac{Q_t}{Q_{rad}} \quad (24)$$

## IV. RESULTS

In this work the Transverse Transmission Line (TTL) method was applied, using double Fourier Transform [11].

Computational algorithms were developed in Matlab and Fortran PowerStation languages.

Fig.4 shows the resonance frequency as function of the patch length, for different critical temperatures. The parameters used are,  $w = 2.5$  mm,  $l = 3.0$  mm,  $\epsilon_{r1} = 8.7209$  (s-polarization),  $\epsilon_{r2} = 1$ ; and the  $(\text{Sn}_5\text{In}) \text{Ba}_4\text{Ca}_2\text{Cu}_{10}\text{O}_y$  [7],[14], at the superconducting temperature of 212 K.

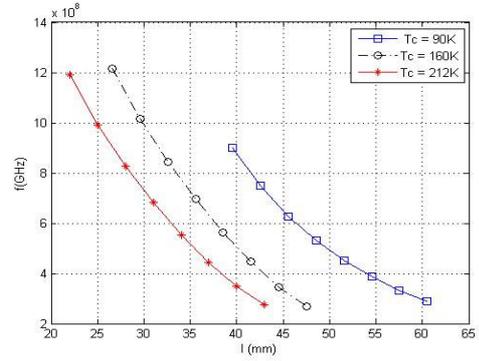


Fig. 4. Resonance frequency in GHz as functions of the patch length, at critical temperatures of 90 K, 160 K and 212 K and PBG substrate with  $\epsilon_{r1}=8.7209$  (s-polarization).

Fig.5 shows the resonance frequency as function of the patch length, for different critical temperatures. The parameters used are,  $w = 2.5$  mm,  $l = 3.0$  mm,  $\epsilon_{r1} = 10.233$  (p-polarization),  $\epsilon_{r2} = 1$ ; and the  $(\text{Sn}_5\text{In}) \text{Ba}_4\text{Ca}_2\text{Cu}_{10}\text{O}_y$  [7],[14], at the superconducting temperature of 212 K.

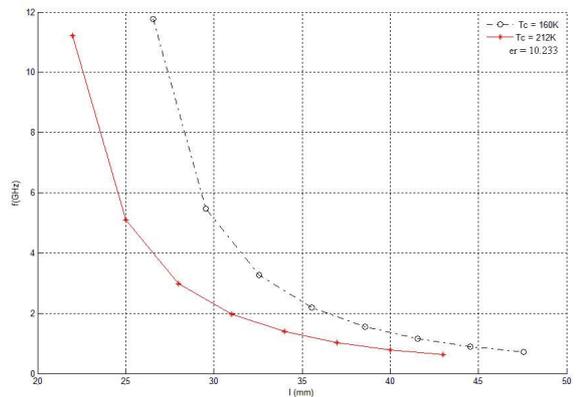


Figure 5. Resonance frequency in GHz as functions of the patch length, at critical temperatures of 160 K and 212 K and PBG substrate with  $\epsilon_r=10.233$ .

Fig. 6 shows the efficiency as function of the frequency for different critical temperature. The parameters used are  $w = 2.5$  mm,  $l = 3.0$  mm,  $\epsilon_{r1} = 10.233$  (p-polarization),  $\epsilon_{r2} = 1$ . With the increase of the frequency, the efficiency increases, and with the increase of the critical temperature the efficiency also increases.

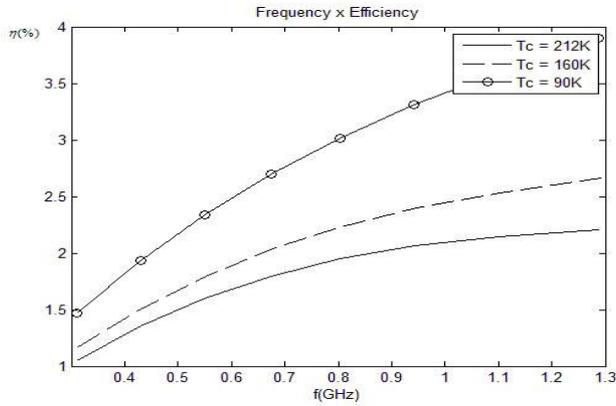


Figure 6. Efficiency as function of the frequency (GHz), of the patch antenna, at critic temperature, of 90 K, 160 K and 212 K,  $\epsilon_{r1} = 10.233$  (p-polarization)

Fig. 7 shows the quality factor as function of the frequency for the superconductor patch with  $T_c = 212$  K. The parameters used are  $w = 2.5$  mm,  $l = 3.0$  mm,  $\epsilon_{r1} = 10.233$ ,  $\epsilon_{r2} = 1$ . With the increase of the frequency, the quality factor decreases.

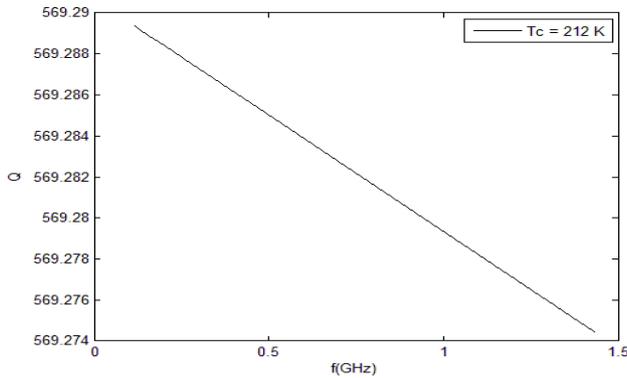
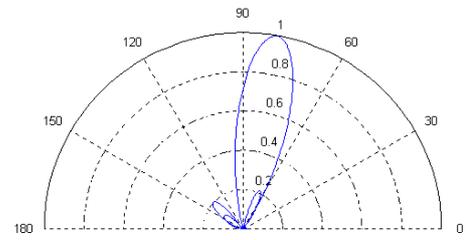


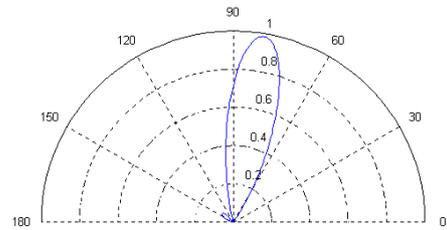
Figure 7. Quality factor as function of the frequency in GHz of the patch superconductor, at critical temperatures of 212 K.

The results in the Fig. 8.a and 8.b shows the pattern fields in E-Plane to a linear array with 4 elements with  $\lambda/2$  for an irradiation angle of  $80^\circ$ , resulting in a phase of  $\beta = 31,25^\circ$ .

Finally the results in Fig. 9.a and 9.b shows the pattern fields in H-Plane for a planar array with 4X4 elements with  $\lambda/2$  for an irradiation angle of  $90^\circ$ .

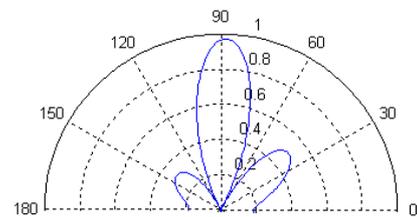


(a)

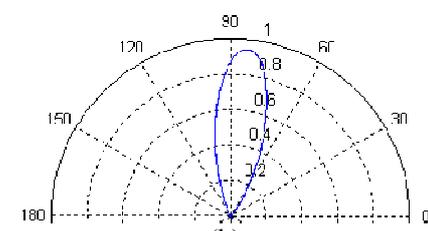


(b)

Figure 8. Pattern fields for a planar array with superconductor patch at  $T_c = 212$  K and  $\theta = 80^\circ$ , for plane-E (a) and plane-H (b).



(a)



(b)

Figure 9. Pattern fields for a planar antenna array with superconductor patch at  $T_c = 212$  K and  $\theta = 90^\circ$ , for (a) plane-E and (b) plane-H.

## V. CONCLUSIONS

The main theories used to explain the phenomenon of superconductivity has been presented at new superconductor materials. The inclusion of superconducting patch was made using the resistive complex boundary condition, and the two fluids model. New numerical results of the efficiency and quality factor were presented. When the frequency increases the efficiency increases, and when the critical temperature increases the efficiency also increases. With the increase of the frequency, the quality factor decreases. The resonant frequency as functions of the antenna array parameters including pattern fields of the E-plane and H-plane, for planar antennas arrays, were also presented. The results are good and show that, when  $T_c$  increase, the dimension of the antenna

reduces. New very high critical temperature material was been presented in this photonic application.

#### ACKNOWLEDGEMENTS

Acknowledgements to CNPQ and CAPES by financial support.

#### REFERENCES

- [1] A.C. Rose-Innes and E.H. Roderik, "*Introduction to Superconductivity*", 2<sup>a</sup> Edition, Pergamon Press, 1978.
- [2] E. A. Linton, "*Superconductivity*", London: Mathuen & Co. LTDA, Ney York: John Wiley & Sons Inc. 1964.
- [3] Zhi-Y. Shen, "High-Temperature Superconducting Microwave Circuits", Artech House, Inc. 1994
- [4] S. Sudhakaran, "Negative Refraction from Electromagnetic Periodic Structures and Its Applications", Doctoral Thesis, Department of Electrical Engineer, Queen Mary University of London, UK, p. 226, Jun. 2006.
- [5] H.C.C. Fernandes, H. M. C. A. Maia and L. M. Caetano, " Simultaneous Photonic Substrate and Superconductor Patch Smart Antennas Array", IMOC2011-SBMO/IEEE MTT-S International Microwave And Optoelectronics Conference, Conf. Proc. 5 pp. CD, paper 90061, Natal-RN, Brazil. Out./Nov. 2011.
- [6] H.C.C. Fernandes, H. M. C. A. Maia, L. M. Caetano and M. P. Sousa Neto, "Microstrip Antennas Array with Very High Critical Temperature", paper N° 71341, ITS'2010/IEEE - International Telecommunications Symposium, Manaus-AM, Br 4 pp.CD, Sept. 2010.
- [7] E.J.Eck, "200K Superconductor Tweaked to 212K", <http://www.superconductors.org/>, Jan. 2009.
- [8] H.C.C. Fernandes and H. M. C. A. Maia, "Superconductor Substrate With Critical Temperature at 212 K For Planar Antenna", Journal of Materials Science and Engineering, USA, David Publishing Company Vol. 6, pp. 78-82. Jul. 2010.
- [9] J. M. Pond, C. M. Krowne and W. L. Carter, "On Application of Complex Resistive Boundary Conditions to Model Transmission Lines Consisting of Very Thin Superconductors", IEEE-MTT, Vol. 37, No 1, pp. 181-189, Jan. 1989.
- [10] Z. Cai and J. Bornemann, "Generalized Spectral-Domain Analysis for Multilayered Complex Media and High-Tc Superconductor Application", IEEE-MTT, Vol. 40, No 12, pp. 2251-2257, Dez. 1992.
- [11] D. Nghiem, J. T. Williams and D. R. Jackson, "A General Analysis of Propagation Along Multiple-layer Superconducting Stripline and Microstrip Transmission Lines", IEEE-MTT, Vol. 39, No 9, pp. 1553-1565, Set.1991.
- [12] H.C.C. Fernandes, R. R. C. França and D. B. Brito, "Asymmetric FinLine and Coupler at Millimeter Waves on PBG Substrate", Journal of Infrared, Millimeter, and Terahertz Waves, V. 32, N° 1, pp. 116-125, Jan. 2011.
- [13] C. A. Balanis, "Antenna Theory Analysis and Design", 2<sup>nd</sup> ed., J. Wiley & Sons, INC, 1997
- [14] H.C.C. Fernandes and H. M. C. A. Maia, "New Antenna with Superconductor at Critical Temperature of 212". IMOC2009-SBMO/IEEE MTT-S International Microwave and Optoelectronics Conference, Belem – PA, Brazil, pp. 197-199, Nov. 2009.
- [15] H.C.C. Fernandes, H. M. C. A. Maia and L. M. Caetano, " Rectangular Microstrip Antennas Linear Array with Superconductor at High Temperature", WORLDCOMP'11 - The 2011 World Congress in Computer Science, Computer Engineering, and Applied Computing, ICWN'11- The 2011 International Conference on Wireless Networks (ICWN'11) , paper N° ICW8555, Las Vegas, Nevada, EUA, 4 pp. CD, July 2011.