Negative Refraction and Spatial Dispersion in Metamaterials

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Abstract- Both negative refraction and spatial dispersion has been observed in metamaterials. A combination of simulated and measured data is used to show the relationship between the negative refraction and spatial dispersion in a split-ring/wire-post Metamaterial. When the spatial modes are excited, three waves propagate through the metamaterial; incident, longitudinal, and transverse waves. The relationship between spatial dispersion and negative refraction along with excitation and control of the spatial modes in a metamaterial will be presented and discussed.

In 1968 Veselago\textsuperscript{1} proposed that negative refraction would occur in a medium when the permittivity ($\varepsilon$) and permeability ($\mu$) were simultaneously negative as shown in Figure 1 (a). This was realized with the advent of an effective media model\textsuperscript{2}, which could define a plasma frequency ($\omega_p$) of a wire array, where $\varepsilon < 0$ for $\omega < \omega_p$, and the development of a model for split-ring-resonators (SRRs)\textsuperscript{3}, which could produce a magnetic resonance where $\mu < 0$ in a frequency band. A SRR/wire-post medium with $\varepsilon < 0$ and $\mu < 0$ was used as the building-block to demonstrate propagation properties of a wave through a slab\textsuperscript{4}, followed by a demonstration of negative refraction\textsuperscript{5} using a wedge shaped prism. Figure 1 (a) shows the simulation of a plane wave propagation through a wedge with $\varepsilon < 0$ and $\mu < 0$ being negatively refracted ($-\theta$) to the same side of the normal exiting the wedge as the incident wave, as predicted by Veselago. Figure 1 (b) shows a simulation of a plane wave propagation through a wedge with $\varepsilon > 0$ and $\mu > 0$ being positively refracted ($+\theta$) to the opposite side of the normal exiting the wedge as the incident wave. Figure 1 (a) and (b) shows the waves exiting the wedges look different than the plane waves on incident the wedges. The change in the appearance of the exiting waves is dependent on the dispersive behavior of the wedge. In Figure 1 (b), the dispersion of the wedge is only frequency dependent $\varepsilon(\omega)$ and $\mu(\omega)$, comparing the waves exiting the wedges in Figure 1(b) and Figure 1 (a) indicates the dispersion of the wedge in Figure 1 (b) is dependent on more than just frequency.

![Negative Refraction](image1.png)  
![Positive Refraction](image2.png)

Figure 1: (a) Shows an incident plane wave propagating through a wedge shaped prism with $\varepsilon<0$ and $\mu<0$ being negatively refracted as it exits the wedge, (b) shows an incident plane wave propagating through a wedge with $\varepsilon > 0$ and $\mu > 0$ being positively refracted as it exits the wedge.

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Metamaterials have been shown to exhibit frequency and spatial dispersion\(^6\). In a frequency and spatially dispersive medium the permittivity \(\varepsilon(\omega, k)\) and permeability \(\mu(\omega, k)\) are dependent on the frequency \(\omega\) and the potential \(k\) which is a function of the periodicity of the lattice in a crystal or the unit cell in a metamaterial. Spatial dispersion is also dependent on the ratio period \((a)\) of the lattice or unit cell to the free space wavelength \((\lambda_0)\) of the incident wave where \(a/\lambda_0 < 1\). In this work, we demonstrate the behavior of the longitudinal and transverse spatial modes in a metamaterial and show their effect on the loss and negative refraction of the medium. To demonstrate the longitudinal and transverse behavior in the metamaterial we ran parametric simulations changing the geometric dimensions of the SRRs and wire-posts while keeping the period of the unit cells constant. Figure 2 below shows a unit cell used in the simulation and the extracted results of \(\varepsilon(\omega, k)\) and \(\mu(\omega, k)\) for 3 unit cell configurations. The plots of Figure 3 (a), (b), and (c) below also show how the spatial dispersion varied in the unit cell as the geometrical dimensions of the SRRs and wire-post were changed. The unit cells described in Figure 3 (a) and (b) were used for fabrication of slab and wedge prism structures to demonstrate the interaction of the longitudinal and transverse spatial waves with the incident wave. The frequency response and polarization behavior of the slabs and refractive behavior of the wedges were measured. Using the simulated and measured results, we were able to show the interaction between the longitudinal and transverse spatial waves with the incident wave propagating in the metamaterial and how the spatial waves affect the losses and negative refraction in the metamaterial.

**Figure 2:** A unit cell for the simulation and extraction of \(\varepsilon\) and \(\mu\) for (a) a unit cell with 2.79 mm OD SRRs and 0.25 mm wire post, (b) the unit cell in (a) with a 0.75 mm wire post instead and (c) a unit cell with a 3.81 mm OD SRR and a 0.75 mm wire post.

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